In this example, only 16 triangular second order finite elements are used to obtain a convergent result.

V. Conclusions

A generalized coupled finite and boundary element method has been shown to be applicable to microwave planar circuit problems. The planar waveguide model is used in developing the technique. The technique takes advantage of the strengths of the finite element and boundary element methods. Thus, it can handle complicated and arbitrarily shaped planar circuits with a small computational overhead. The validity of the method was confirmed by comparing the CFBM results, either with published results or with experimental results. The performance of a Y-junction circulator with an equilateral triangular ferrite post was also investigated. For all the numerical examples, the power conservation condition has been found to be satisfied to an accuracy of $\pm 10^{-5}$ to $\pm 10^{-4}$ within the frequency band of the dominant mode.

Acknowledgment

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A Modified Transverse Resonance Method for the Analysis of Multilayered, Multiconductor Quasiplanar Structures with Finite Conductor Thickness and Mounting Grooves

Jun-Wu Tao

***Abstract*—A modified transverse resonance method is presented for analyzing generalized multilayered, multiconductor quasiplanar structures with practical parameters such as finite conductor thickness and mounting grooves. Recurrence relations are obtained by using network theory, for obtaining the overall transverse equivalent network, while the discontinuity involving finite thickness metal sheet and mounting groove is carried out by field-theory based multimodal variational formulation. The frequency behavior of propagating, evanescent and complex modes are obtained for several commonly used quasiplanar lines, showing good agreement with published results. Furthermore, the leaky-wave study is carried out for open structures, since the open condition can be included in this formulation without difficulties.**

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I. Introduction

Various planar and quasiplanar structures play a determinant role in the realization of MIC's and MMIC's, which found growing interest in microwave and millimeter-wave subsystems design [1], [2]. Many rigorous, full-wave analysis techniques have been developed for their characterization, among which the most popular will be the spectral domain approach [3], [4], the integral equation technique [5]–[8], and various mode-matching techniques [9]–[12]. It has been shown that the spectral domain approach, as well as the integral equation technique, are numerically efficient. On the other hand, the mode-matching techniques are more versatile since the practical parameters of real quasiplanar structures, such as the metallization thickness and the mounting grooves, can be easily taken into account; nevertheless, more numerical effort will be paid because of relatively large matrix involved.

We present here another modified transverse resonance technique. Contrary to the existing one in which the problem is formulated entirely by the field theory [9], [11], we use both the field and network theories, as in the classical transverse resonance method [13]. By considering the generalized quasiplanar structure as cascaded parallel-plate waveguides, each homogeneous region will be characterized by a transmission matrix, instead of the TEM line in [13], and the junction between two parallel-plate waveguides characterized by a multiport, instead of a shunt admittance. The characteristic equation can then be easily found by applying the resonance condition. The solution is obtained only by applying the well known and easy to use network theory, except for the reduced impedance matrix of parallel-plate waveguide junctions which will be achieved by a rigorous multimodal variational method [14]. Furthermore, as we can choose the size of each impedance matrix, that is, the number of coupling modes in each parallel-plate waveguide, without affecting the accuracy of remaining matrix elements, as explained in [14], the resultant matrix size can be much smaller than the number of eigenmodes considered, providing a convenient way to accelerate the numerical computations. The classical transverse resonance method is then a particular case of this formulation when only the dominant TEM mode is considered. A quasiplanar structure simulation program has been developed on a personal computer, and applied to several commonly used planar and quasiplanar waveguides by considering both the metallization thickness and the mounting groove. Original results such as the complex and backward modes in the suspended microstrip are presented, as well as the leaky modes in the open quasiplanar structure which is very useful in the novel-type millimeter-wave antenna design [15], [16]. The latter has been compared to the published results concerning a leaky-wave microstrip antenna [15], showing good agreement.

II. Modified Transverse Resonance Method

Fig. 1(a) shows a multilayered, multiconductor quasiplanar guiding structure, with its equivalent transverse network model in Fig. 1(b). The reduced impedance matrix of each multiport, which

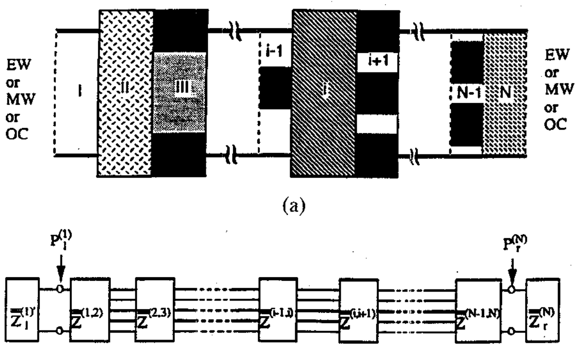


Fig. 1. (a) Cross-section of a generalized quasiplanar structure; the two terminal planes can be either electric wall (EW), magnetic walls (MW) or open (OC); (b) Equivalent transverse network model.

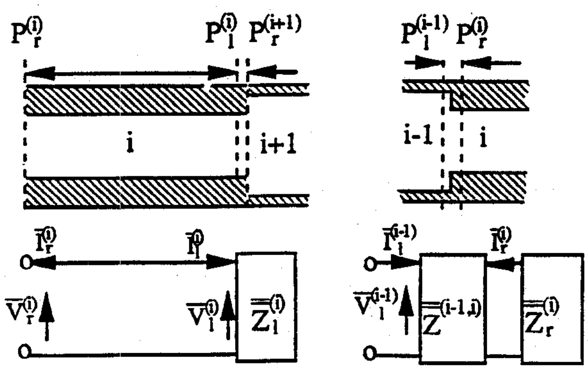


Fig. 2. (a) Impedance matrix seeing through a section of line; (b) Impedance matrix seeing through a multiport.

characterizes the junction between two parallel-plate waveguides, will be obtained by applying the field theory based multimodal variational formulation [14]. The characteristic equation is yielded by using the transverse network resonant condition, in which both transmission and impedance matrices will be used.

A. Reduced Impedance Matrix Associated with a Parallel-Plate Waveguide Junction

The electric and magnetic fields transverse to the x-direction will be expanded over the TE_x and TM_x basis in each region as follows, by omitting the common factor $e^{-\gamma z}$

$$\boldsymbol{E}_t^{(i)} = \begin{bmatrix} E_y \\ E_z \end{bmatrix}^{(i)} = \sum_n v_n^{(i)} / \sqrt{|y_n^{(i)}|}\, \boldsymbol{e}_{tn}^{(i)},$$

$$\boldsymbol{J}_t^{(i)} = \begin{bmatrix} -H_z \\ H_y \end{bmatrix}^{(i)} = \sum_n i_n^{(i)} y_n^{(i)} / \sqrt{|y_n^{(i)}|}\, \boldsymbol{e}_{tn}^{(i)}. \tag{1}$$

Boldface italic letters are used here for space vectors. $v_n^{(i)}$ and $i_n^{(i)}$ correspond to the sum and difference of the x-direction incident and reflected wave amplitude of the TE_x and TM_x eigenmodes in the ith region, with $\boldsymbol{e}_{tn}^{(i)}$ and $y_n^{(i)}$, the electric mode function and the mode admittance, are defined respectively by

$$e_{yn}^{(i)} = \sqrt{\epsilon_n / w^{(i)}} \cosh \tau_n^{(i)} \cos k_{yn}^{(i)} y,$$

$$e_{zn}^{(i)} = \sqrt{\epsilon_n / w^{(i)}} \sinh \tau_n^{(i)} \sin k_{yn}^{(i)} y, \tag{2}$$

$$y_n^{(ih)} = k_{xn}^{(i)} / \omega\mu_0, \quad y_n^{(ie)} = \omega\epsilon_0\epsilon_r^{(i)} / k_{xn}^{(i)} \tag{3}$$

with $k_{xn}^{(i)2} = k_0^2 \epsilon_r^{(i)} + \gamma^2 - k_{yn}^{(i)2}$, $k_{yn}^{(i)} = n\pi / w^{(i)}$, $\epsilon_0 = 1$, $\epsilon_n = 2$, for $n \neq 0$. The superscripts h and e correspond, respectively, to TE_x and TM_x eigenmodes. Expressions for other parameters will be given in the Appendix.

Since the above mode function basis $\boldsymbol{e}_{tn}(\gamma)$ is orthogonal to its adjoint defined by $\boldsymbol{e}_{tn}^{+}(\gamma) = \boldsymbol{e}_{tn}(-\gamma)$, the characterization of junctions between parallel-plate lines may be carried out by the multimodal variational method [14]. The reduced impedance matrix corresponds to the accessible ports will be given, according to [14], by

$$(\bar{\bar{Z}})_{mn}^{(i,i+1)} = -j\sqrt{|y_m / y_n|}\, y_n \bar{\bar{\Gamma}}_{mn}^{(i,i+1)}. \tag{4}$$

A detailed development will be given in the Appendix. It is shown that the accuracy of each element in (4) depends on the number of considered eigenmodes, and the trial aperture electric fields in which the edge condition can be easily included. The size of the reduced impedance matrix depends only on the number of coupling modes which may be very small compared with the total number of eigenmodes used. Furthermore, the impedance matrix formulation allows the use of a different number of eigenmodes in the ith and $(i + 1)$th region, contrary to the transmission matrix formulation. This will be very useful for achieving a faster numerical convergence.

B. Recurrence Relations for Cascaded Networks

Two situations exist when cascading multiple transverse networks as shown in Fig. 2 (a)–(b). When the ith region is terminated by a known charge at its left end, characterized by a reduced impedance matrix $\bar{\bar{Z}}_l^{(i)}$ (Fig. 2(a)), the reduced impedance matrix looking from the right end will be given by using the ith region transmission matrix

$$\bar{\bar{Z}}_r^{(i)} = (\bar{\bar{T}}_1^{(i)} \bar{\bar{Z}}_l^{(i)} + \bar{\bar{T}}_2^{(i)})(\bar{\bar{T}}_2^{(i)} \bar{\bar{Z}}_l^{(i)} + \bar{\bar{T}}_1^{(i)})^{-1} \tag{5}$$

with $\bar{\bar{T}}_1^{(i)}$, $\bar{\bar{T}}_2^{(i)}$ two diagonal matrices in which the lth diagonal element corresponds respectively to $\cos(k_{xl}^{(i)} L^{(i)})$ and $j \sin(l_{xl}^{(i)} L^{(i)})$. For the case shown in Fig. 2(b), the reduced impedance matrix looking through a multiport characterized by $\bar{\bar{Z}}^{(i-1,i)}$ and terminated by $\bar{\bar{Z}}_r^{(i)}$ will be given by

$$\bar{\bar{Z}}_l^{(i-1)} = \bar{\bar{Z}}_{11}^{(i-1,i)} - \bar{\bar{Z}}_{12}^{(i-1,i)} (\bar{\bar{Z}}_r^{(i)} + \bar{\bar{Z}}_{22}^{(i-1,i)})^{-1} \bar{\bar{Z}}_{21}^{(i-1,i)}. \tag{6}$$

C. Generalized Resonance Condition and Characteristic Equation

When applying the recurrence relations (5) and (6), we begin from the last layer of the general quasiplanar structure, and the terminating conditions may be expressed by a diagonal reduced impedance matrix located at $P_r^{(N)}$ with its lth element given by

$$(\bar{\bar{Z}}_r^{(N)})_{ll} = \begin{cases} -j \tan(k_{xl}^{(N)} L^{(N)}), & \text{EW (Electric Wall)} \\ -1 & \text{OC (Open Condition)} \\ j \cot(k_{xl}^{(N)} L^{(N)}), & \text{MW (Magnetic Wall)} \end{cases} \tag{7}$$

By using alternatively (5) and (6), we obtain $\bar{\bar{Z}}_l^{(1)}$. The generalized resonance condition will be given by

$$\bar{\bar{Z}}_l^{(1)'} + \bar{\bar{Z}}_l^{(1)} = 0 \tag{8}$$

with $\bar{\bar{Z}}_l^{(1)'}$ obtained in the same manner as for $\bar{\bar{Z}}_r^{(N)}$. The non-trivial solution is yielded by equating the determinant of $\bar{\bar{Z}}_l^{(1)'} + \bar{\bar{Z}}_l^{(1)}$ to zero, which is the characteristic equation of the generalized quasiplanar structure.

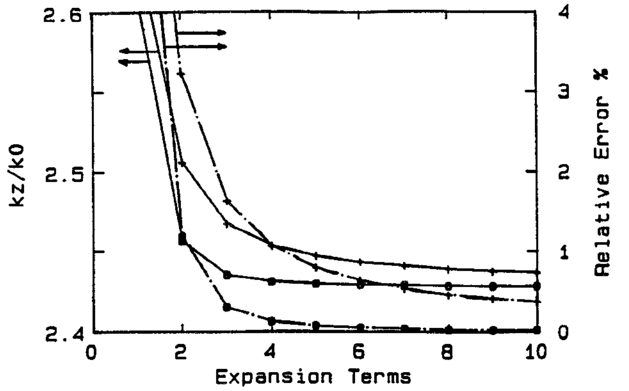
III. Practical Applications to Quasiplanar Structures

As we can see in the Appendix, the reduced impedance matrix depends on the choice of three parameters: the number of eigenmodes in both parallel-plate lines involved, N_1 and N_2, and the aperture electric field expansion terms, M, with or without considering the metal sheet edge effect. Additionally, the analysis of the overall structure depends also on the number of coupling modes in each region. The influence of both parameters has been illustrated by analyzing a microstrip-like transmission line [5]. Fig. 3(a) shows the variation of the normalized propagation constant versus the expansion terms M. The computation has been carried out by using respectively 20, 30 and 50 eigenmodes in both dielectric and air-filled regions, no significant difference (less than 0.3%) has been observed for these three cases. Fast convergence has been obtained with an eigenfunction basis considering the edge conditions. The variation of the normalized propagation constant versus the number of coupling modes in the strip region, where 10 eigenmodes are considered, has been given in Fig. 3(b) for respectively 100 μm and 10 μm strip thickness cases. For both cases only 1% error will result if $\frac{1}{3}$ of eigenmodes are considered accessible, this means the size of corresponding impedance matrix is only $\frac{1}{3}$ of the original one.

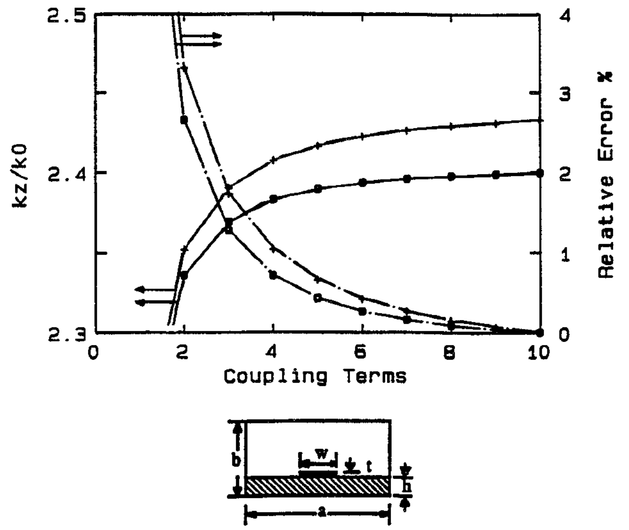
Several commonly used quasiplanar structures such as the shielded microstrip line, uni- and bilateral finline, coplanar waveguide has been studied by the present method, showing good agreement with other full-wave analysis results. When applying this method to the suspended microstrip with finite strip thickness for both propagating and evanescent cases, we have observed the mode disappearance [7] for several pairs of evanescent modes, which predicts the existence of complex modes. The latter has been obtained by scanning the complex plane of γ, and the results are shown in Fig. 4.

The results of [11] are also given in the same figure, showing good agreement for most of the modes, except for those derived from the complex modes, for which a significant difference can be observed near the cutoff. The backward-wavemodes, such as the 4th even-mode, have also been observed.

It is shown that many open printed transmission lines can support leaky-wave modes with suitable excitation and appropriate choice of structure sizes [16]. This is very useful in scannable millimeter-wave antenna design in which manufacture becomes an important criterion. Most of these lines can be represented by the generalized quasiplanar structure of Fig. 1(a) with the open condition (7). When infinite dielectric substrate is considered, as for the leaky-wave microstrip antenna [15], we can always introduce two lateral metal planes at a certain distance. The complex leaky-wave constant obtained in this way has been given in Fig. 5, and compared with the results of [15], showing good agreement in the frequency range in which the structure can be used for antenna design. The bilateral finline has also been studied with one of the lateral walls removed, and the results for the first odd-mode are shown in Fig. 6, compared with those of the shielded structure. We can see that no significant difference exists in the higher frequency range, but when the frequency decreases, the odd-mode becomes leaky. Generally speaking, the design of a leaky-wave guide can be carried out only according to the shielded structure analysis, since the working frequency range is defined by $0 < \beta/k_0 < 1$, with β being the real phase constant.



(a)



(b)

Fig. 3. Convergence studies for a microstrip-like transmission line [5]. The normalized propagation constant and the relative error (compared with the quasi-TEM solution) versus: (a) Aperture field expansion terms with normal (+ +) and modified (○ ○) eigenfunction basis; (b) Coupling terms in the strip region with a strip thickness of 10 μm (+ +) and 100 μm (○ ○); $w = h = 1.27$ mm, $a = b = 12.7$ mm, $\epsilon_r = 8.875$.

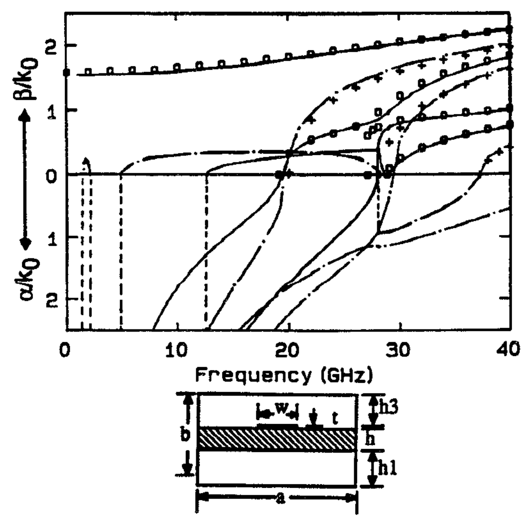


Fig. 4. Propagating, evanescent and complex modes of a suspended microstrip line; Our results for even (——) and odd (– –) modes compared with those of [11] (○, +); WR28 waveguide with $w = 1$ mm, $h_2 = 0.635$ mm, $h_1 = h_3 + t$, $t = 5$ μm, $\epsilon_r = 9.6$.

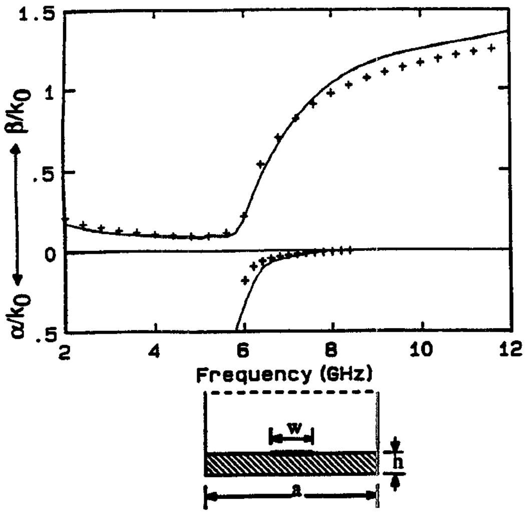


Fig. 5. Leaky-wave mode of a microstrip leaky-wave antenna; Our results (——) compared with data taken from [15] (+ +). Dimensions used: $w = 15$, $h = 0.794$, $a = 45$ (mm); $\epsilon_r = 2.32$.

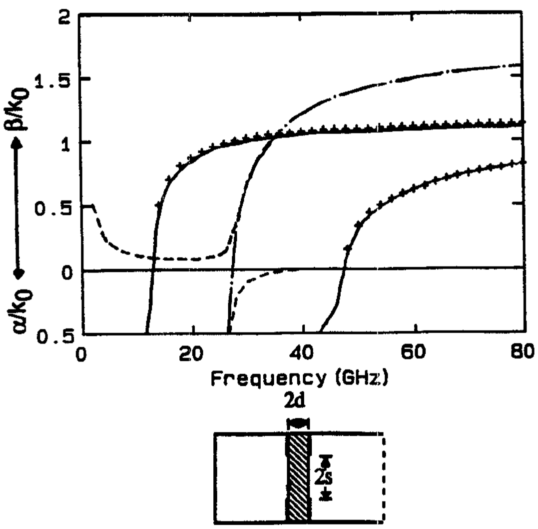


Fig. 6. Dispersion behavior of a bilateral finline. Even (——) modes compared with those of [4] (+ +); Odd modes with (– –) and without (– · –) open condition. WR28 waveguide with $2s = 0.5$, $2d = 0.125$ (mm), $\epsilon_r = 3$.

IV. Conclusion

This paper presents a modified transverse resonance method for analyzing generalized quasiplanar structures with practical parameters such as finite conductor thickness and mounting grooves. The computation of the frequency behavior of propagating, evanescent and complex modes have been carried out for several commonly used quasiplanar lines, and good agreement with published results has been obtained. Furthermore, as the open condition can be easily taken into account by using this formulation, the leaky-wave modes for open quasiplanar structures have also been studied, giving useful design information.

Appendix

The quantities appearing in (2)

$$\begin{aligned}&\text{*for TE case: } \cosh \tau_n^{(i)} = \gamma / Q_n^{(i)},\ \sinh \tau_n^{(i)} = -k_{yn}^{(i)}/Q_n^{(i)}\\ &\text{*for TM case: } \cosh \tau_n^{(i)} = k_{yn}^{(i)}/Q_n^{(i)},\ \sinh \tau_n^{(i)} = -\gamma/Q_n^{(i)}\end{aligned} \qquad \text{(A1)}$$

where $Q_n^{(i)2} = k_{yn}^{(i)2} - \gamma^2$, and $\gamma = \alpha + j\beta$ is the complex propagation constant. $w^{(i)}$ denotes the height of a parallel-plate waveguide.

Matrix elements $\overline{\overline{\Gamma}}_{mn}^{(i,i+1)}$ *in (4)*

$$\overline{\overline{\Gamma}}^{(i,i+1)} = \overline{\overline{F}}^t \overline{\overline{Q}}^{(i,i+1)-1} \overline{\overline{F}}$$

with

$$\overline{\overline{Q}}_{mn}^{(i,i+1)} = (-j)\langle g_m, \hat{Y}^{(i,i+1)} g_n \rangle,$$

$$\overline{\overline{F}}^t = [\overline{\overline{U}}^{(i)} \overline{\overline{U}}^{(i+1,1)}] \cdots \overline{\overline{U}}^{(i+1,N)}]^t$$

$$(\overline{\overline{U}}^{(i)})_{ml} = \langle g_m, e_{tl}^{(i)} \rangle, \qquad l = 1, 2, \cdots, K^{(i)};$$

$$(\overline{\overline{U}}^{(i+1,q)})_{ml} = \langle g_m, e_{tl}^{(i+1,q)} \rangle, \qquad l = 1, 2, \cdots, K^{(i+1,q)};$$

$$\hat{Y}^{(i,i+1)} = \sum_{k=K^{(i)}+1}^{\infty} y_k^{(i)} \hat{P}_k^{(i)} + \sum_{q=1}^{N} \sum_{l=K^{(i,q)}+1}^{\infty} y_l^{(i,q)} \hat{P}_l^{(i,q)}; \qquad \text{(A2)}$$

$$\hat{P}_k^{(i)} F = e_{tk}^{(i)} \langle e_{tk}^{(i)}, F \rangle = e_{tk}^{(i)} \int (e_{tk}^{(i)+} F)\, dS$$

$K^{(i)}$ corresponds to the number of coupling eigenmodes in the ith region, and $G = \{g_n\}$ denotes the eigenfunction basis for the aperture electric field expansion in an N-furcated parallel-plate waveguide given by

$$G = [e_{tl}^{(i+1,1)} \cdots e_{tp}^{(i+1,1)} \cdots e_{tl}^{(i+1,N)} \cdots e_{tq}^{(i+1,N)}] \qquad \text{(A3)}$$

One should note that (A3) will be truncated during the numerical process. When the edge condition is considered, the following basis will be used instead of (A3),

$$G\,[\overline{G}_y \overline{G}_z] \qquad \text{(A4)}$$

with

$$g_{yn}^{(i+1,q)} = e_{yn}^{(i+1,q)} \Delta y^{-\nu}, \quad g_{zn}^{(i+1,q)} = e_{zn}^{(i+1,q)} \Delta y^{-\nu}$$

$$\Delta y = (w^{(i+1,q)}/2)^2 - (y - t^{(i+1,q)} - w^{(i+1,q)}/2)^2$$

ν will be $\frac{1}{2}$ for the zero thickness strip case, and $\frac{1}{3}$ for the finite thickness strip. By examining (A2)–(A4) and (2), one can see that all integrals will be analytic and independently of γ.

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Dispersion and Leakage Characteristics of Coplanar Waveguides

Jeng-Yi Ke, I-Sheng Tsai, and Chun Hsiung Chen

Abstract—**The spectral-domain approach is utilized to discuss the dispersion and leakage phenomenon in a coplanar waveguide structure caused by the substrate surface wave. In this study, the effective dielectric constant and the attenuation constant due to surface wave leakage are presented and discussed in detail.**

I. Introduction

Recently the coplanar waveguide structure receives increased attention due to its potential applications in millimeter wave spectrum. With all three conductors on the same side of the substrate, the coplanar waveguide is easy in adaptation to active and passive components in shunt and series configurations and hence becomes a useful component of millimeter-wave integrated circuits.

The coplanar waveguide structure was proposed by Wen [1] as a transmission medium in microwave circuits. Its dispersion characteristics were studied, using the full-wave analyses such as spectral-domain approach [2] and hybrid approach [3].

The possibility of leakage in coplanar waveguide structure through substrate surface-wave modes was discussed and estimated by a simplified theory based on reciprocity [4], [5]. Leakage to substrate surface-wave modes was also observed in other structures such as the coplanar stripline [6], the slot line [7], [8], the microstrip on an anisotropic substrate [9], and the conductor-backed coplanar waveguide [5]. Recent leakage study on coplanar waveguides of finite and infinite widths revealed several interesting behaviors such as sharp and deep minima and narrow sharp peaks [10]. Since power leakage through surface waves may produce undesired cross talk and package effects, there is a need of detail leakage analysis for the coplanar waveguide structure.

In this study, the spectral-domain analysis will be utilized to discuss the leakage phenomenon in an open coplanar waveguide structure caused by the substrate surface wave. The dispersion and leakage characteristics of the coplanar waveguide will then be discussed in detail, which include typical numerical results such as the effective dielectric constant and the attenuation constant due to surface wave leakage.

II. Spectral-Domain Analysis

Consider the coplanar waveguide structure (insert of Fig. 1) with strip width w, slot width, s, and a substrate of thickness h and relative dielectric constant ϵ_r. It is assumed that all field quantities are of the form $\exp\,[j(\omega t - k_z z)]$. To conduct the spectral-domain analysis, the Fourier transformation pair is introduced as

$$\tilde{A}(k_x) = \int_{-\infty}^{\infty} A(x)e^{-jk_x x}\,dx$$
$$A(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{A}(k_x)e^{jk_x x}\,dk_x. \tag{1}$$

Then a relation which relates electric currents ($\tilde{J}_z$, $\tilde{J}_x$) to electric fields ($\tilde{E}_z$, $\tilde{E}_x$) in the spectral domain can be established [11]

$$\begin{pmatrix}\tilde{J}_z\\ \tilde{J}_x\end{pmatrix} = \begin{pmatrix}\tilde{G}_{zz} & \tilde{G}_{zx}\\ \tilde{G}_{xz} & \tilde{G}_{xx}\end{pmatrix}\begin{pmatrix}\tilde{E}_z\\ \tilde{E}_x\end{pmatrix}. \tag{2}$$

Here $\tilde{G}_{zz}$, $\tilde{G}_{zx}$, $\tilde{G}_{xz}$, and $\tilde{G}_{xx}$ are the transformed Green's functions whose poles may be identified with the characteristic surface wave modes of the dielectric slab with back-side metallization.

In this analysis, the tangential electric fields on the slot are expanded as

$$E_z(x) = \sum_{\nu} C_z^{\nu}\,\Phi_z^{\nu}(x)$$
$$E_x(x) = \sum_{\nu} C_x^{\nu}\,\Phi_x^{\nu}(x), \tag{3}$$

where C_z^{ν} and C_x^{ν} are unknown coefficients to be determined and $\Phi_z^{\nu}(x)$ and $\Phi_x^{\nu}(x)$ are known basis functions as suggested by [12]. By applying the Galerkin's procedure in the spectral domain, the following matrix equation can be derived

$$[Z_{ij}^{\nu\mu}][C] = 0 \tag{4}$$

where

$$[C] = [C_z^{\nu} C_x^{\nu}]^t$$
$$Z_{ij}^{\nu\mu} = \int_{-\infty}^{\infty} \tilde{\Phi}_i^{\nu}\,\tilde{G}_{ij}^{\nu\mu}\,\tilde{\Phi}_j^{\mu}\,dk_x, \qquad i, j \in \{x, z\}. \tag{5}$$

The propagation constant k_z is then obtained by requiring the determinant of the Z-matrix be zero, and the effective dielectric constant $\epsilon_{\text{eff}} = (k_z/k_0)^2$ can be achieved.

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